Getting hooked on primes

Franz Schlindwein uses Izak9 for a deep dive into prime numbers

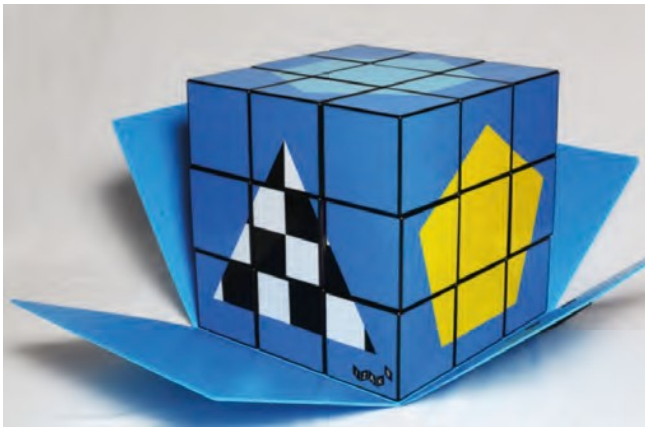
The teaching and learning of prime numbers usually involves the following:

- One is not prime;
- Although even, two is prime;
- A prime number has only 2 factors, itself and one;
- and the use of the Sieve of Eratosthenes to introduce an element of discovery.

In most cases pupils will very soon arrive at a text book, app or worksheet asking them to either list prime numbers or identify them from a list, as a mechanism for helping them become familiar with the set of numbers, maybe remember the first few and help inform them how to work out the rest.

We thought we would try a different approach. We wanted to create an opportunity for pupils to build a relationship with this set of numbers by discovery, that would help them to identify with their properties, as well as attaching hooks to the numbers, to help them retain knowledge of the set and its properties.

Izak9



Izak9 is a set of twenty-seven cubes, presented to pupils as shown on the previous page. Pupils open the box and are transfixed by the wonder revealed before them.

The device creates a learning environment that is open, visual, shared, rich and curious.

Once pupils open the box of cubes and share them out, every pupil in the class becomes a member of a 4/5–person team, with nine cubes per team.

An online portfolio of support material is opened on the screen at the front of the room by the teacher and two robot characters, Abacus (pictured below) and Helix, facilitate the delivery of maths problems with their identical set of virtual cubes. Pupils work together with the real cubes, to solve the problems and answer any additional questions posed.

Odd one out

One of the tasks available in our Izak9 user area is Odd one out. Our two robot facilitators introduce the task with no mention of the word prime.

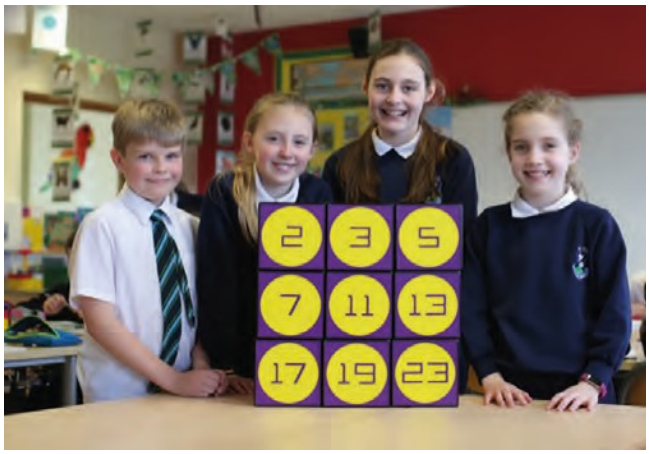
Pupils are simply asked to identify the yellow circle that is on every cube and arrange them in a three by three array in order, with the lowest in the top left corner, increasing in an order shown, to the largest in the bottom right.

As with every Izak9 task, Abacus and Helix ask pupils to: Plan, Do, Review and Make Sense.

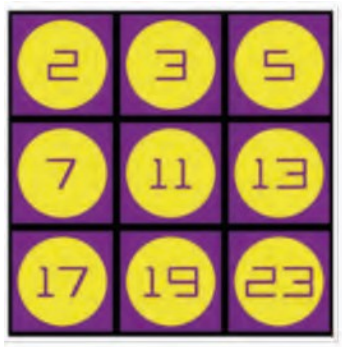
Plan what your team is going to do and who is going to do it.
Do the task together.
Review how the team performed on the task.
Ask: Does our answer **Make Sense**?

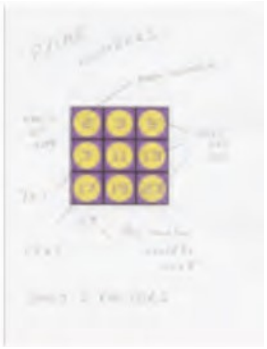
Pupils work in small 4/5 person teams and build the wall of cubes shown above right (*video link shows task posed by Abacus and Helix and pupils building the wall* https://www.izak9.com/work).

Once all the pupils have built their wall, we have a conversation about the numbers on display. The conversation usually revolves around the properties of the numbers with most of what is contained in



the first paragraph of this article, being mentioned. Everybody arrives at the conclusion that these numbers are prime.





At this point, we often get pupils making unpredictable observations and it isn't possible to reference these now. When these situations do occur, we try to pursue any investigative opportunities that arise.

Additional observations aside, the first question usually posed is this: What is the sum of the numbers in the yellow circles?

I, as an adult who had been taught maths in a very traditional way as a child, on my first attempt at answering my own question, set about these numbers in increasing order, and after a long laborious effort, arrived at the correct answer of 100. I was delighted. I had no idea that the first nine prime numbers totalled 100 and I knew this would give me an abundance of questioning opportunities with this wall of cubes. What I hadn't reckoned on, was what kids would do.

The first time I asked this question to a group of children, a hand went up after 3 seconds. Then another, less than 5 seconds later. I couldn't believe it.

The first child got to her feet and pointed to the cubes. "Seven plus twenty-three is thirty, eleven and nineteen makes thirty, thirteen and seventeen make thirty and two, three and five make ten. Answer is one hundred."

The second child got up. “I don’t like that way.”

“What way do you like?” I asked.

“I like this way” and pointing to each cube in turn, said, “two, three and five make ten. Seven and thirteen make twenty. Eleven and nineteen make thirty and seventeen and twenty-three make forty. Ten plus twenty plus thirty plus forty equals one hundred.”

This is the point when I knew we were on to something with these yellow circles.

After many hundreds of sessions with many thousands of pupils between then and now, we continue to develop this task.

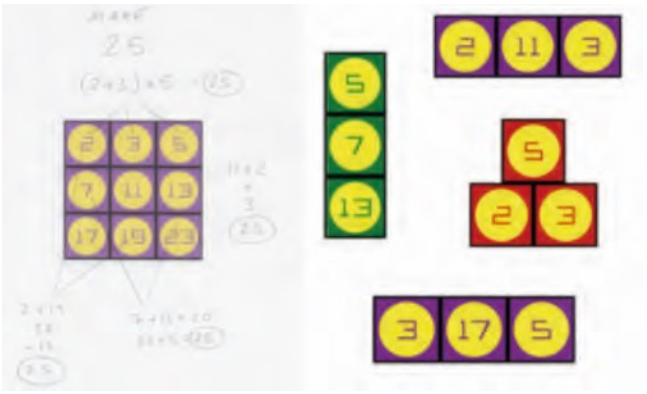
The next thing we ask is this: Can you make 25 using any 3 numbers from the wall once only and any combination of the four number operations?

Children come up with solutions very quickly. They are free to approach the task any way they choose. Some pupils are happy working from the array they have already built and some prefer to remove the cubes and manipulate them into lines or towers, as can be seen in the diagram beside the annotated array.

Examples:

$13 + 7 + 5$ $\quad$ $3 + 17 + 5$

$2 \times 11 + 3$ $\quad$ $(2 + 3) \times 5$



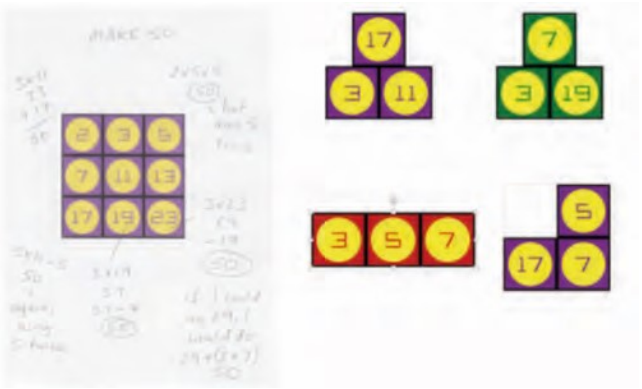
These responses give us the chance to talk about BIDMAS/BODMAS and the need, or not as the case may be, for brackets. We ask the children are these too easy. They say “yes”. So, we move on. Can we do the same task again, but this time, make 50?

This proves a little more challenging. There are normally six groups of about five children, operating in the classroom. The pupils are told that we need a different answer from each group, so they are aware that they will need more than one answer, just in case another group puts forward their solution. Again, they can manipulate the cubes in any manner they choose to help them.

Here are examples of what we get.

$3 \times 11 + 17$ $\quad$ $(3 + 7) \times 5$

$3 \times 19 - 7$ $\quad$ $(17 - 7) \times 5$



Whilst there is no doubt that these responses are of value and great for nurturing mental maths skills and developing strategies, it is the observations we get from children when feeding back their approaches to the other groups, that transpire to be of most value.

Examples are:

“We all do a lot of adding, multiplying and subtracting, but nobody is dividing.”

“We tried doubling numbers and then adding or subtracting a number to make 50, but this kept giving us an odd number. There is only one even number and we had already used that to double.”

“I tried multiplying by 5 to get a number bigger than 50 and then subtract, but there is only one multiple of 5, so I had to try something else.”

“I squared five to try and get an odd number before doubling, but I had to use 3 different numbers and couldn’t use the five twice.”

“I used the prime number that comes next, after the wall and did $29 + 3 \times 7$.”

If we consider these observations:

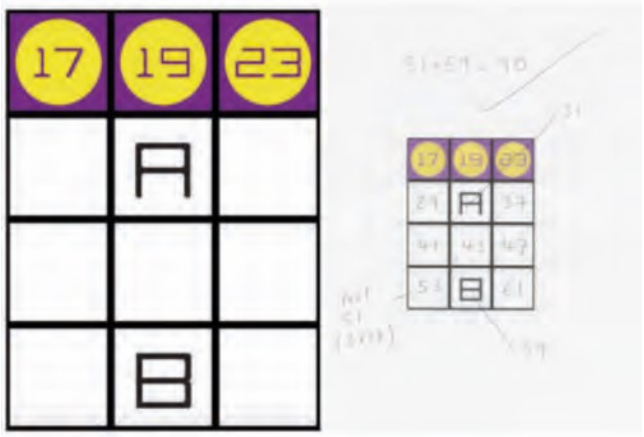
Pupils notice that there is no division happening and have said that this is the case because each number doesn’t have any factors to divide by. Division offers no help in trying to solve the problem.

Doubling a number or doubling the sum/difference of two numbers means you can’t add or subtract to get your answer. This is due to two being the only even number.

There is no other multiple of five, because there are no multiples of any prime number in the set of prime numbers.

Some pupils have worked out the pattern continues and want to consider numbers beyond those exhibited on the wall.

The latter observation opens a door. Once a pupil mentions the possibility of using numbers beyond those on display, we can now develop this conversation and pose an extension to the task. When we get the opportunity to discuss what happens beyond the wall, we can present another question, such as: If the pattern continues beyond the bottom row of your wall, what is the sum of A and B?



The pupils are now investigating the pattern based on the properties of number and the discussions and debates we have had up to this point. They are also solving a problem as opposed to simply listing numbers. Once they have completed the task in their groups, each group has an opportunity to feed back in turn.

Further extension

An example of a further extension is shown below, where the pupils continue the pattern, to solve another problem: Which four consecutive prime numbers add to make two hundred and twenty?

Pupils discuss this with Plan, Do, Review and Make Sense, as this permeates through every task they approach. Some pupils try a trial and improvement method whereby they take 4 consecutive primes and generate a total, moving up and down until they reach their target. Others realise quickly that if they add two of the larger primes, they will get an even number. So, they look for two consecutive pairs of numbers, with two even number totals that will give 220.

Then we get the groups that consider the mean of these numbers and work out that two hundred and twenty divided by four gives fifty-five. They then identify the prime numbers that lie either side of fifty-five and arrive at the solution quite quickly. Any which way, the result is 47 + 53 + 59 + 61 = 220 and the groups get the chance to present their findings to each other.

When we look at this lesson or series of lessons, depending on the parameters involved in teaching the class in front of you, at a minimum level, we have pupils being able to access a visual stimulus of yellow circles that may help them remember what primes are and possibly trigger them to recollect what happened during their investigation.

At a higher level, we have delved deeper into the properties of prime numbers and discovered, via linking with the number strand of the curriculum and in some cases the data handling strand, that patterns exist within this set of numbers.

Perhaps the richest learning to come from this experience is that pupils have shown tenacity and resilience to overcome the limitations imposed on them by the properties of primes to solve problems. This surely provides them with a new set of hooks that can be used to hold on to what they have learned about this set of numbers while they move through from primary to secondary phases.

For more information on **Izak9** please visit **www.izak9.com** or contact **Franz Schlindwein**, creator of the device, on **franz@izak9.com**